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Quantum statistical effects on parton distribution scaling behaviour

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Abstract

Starting from the old idea that Fermi statistics for quarks play a fundamental role to explain some features of hadron structure, we study the modification of the scaling behaviour of parton distributions due to quantum statistical effects. In particular, by following an interesting formal analogy which holds between the Altarelli-Parisi evolution equations, in leading-log approximation, and a set of Boltzmann equations, we generalize the evolution equations to take into account Pauli exclusion principle and gluon induced emission.

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Deep inelastic experiments seem to be an inexhaustible source of information on the hadronic structure and continue to considerably improve our understanding of strong interaction dynamics. A measurement of proton and neutron $F_2(x)$ structure function performed by the NMC Collaboration at CERN [1] suggests a rather large $SU(2)$ flavour breaking in the sea quark [2]. In particular they have obtained a determination, at very small x, for the difference

$$\mathcal{I}_G(x) = \int_x^1 \frac{dy}{y}[F_2^{\mu p}(y) - F_2^{\mu n}(y)] = \frac{1}{3}\int_x^1 dy\ [u(y) + \bar{u}(y) - d(y) - \bar{d}(y)] \quad , \tag{1}$$

finding $\mathcal{I}_G(0.004) = 0.227 \pm 0.007$. Thus, by extrapolating down to $x = 0$, they have estimated

$$\mathcal{I}_G(0) = 0.240 \pm 0.016 \quad . \tag{2}$$

This result represents a relevant violation of the Gottfried sum rule [3], which would predict $\mathcal{I}_G(0) = 1/3$. Moreover, from (2) we get

$$\bar{d} - \bar{u} = \int_0^1 dx\ [\bar{d}(x) - \bar{u}(x)] \sim 0.14 \quad . \tag{3}$$

However, the inequality $\bar{d} > \bar{u}$ was already argued many years ago by Field and Feynman [4] on pure statistical basis. They suggested that in the proton the production from gluon decays of $u\bar{u}$-pairs with respect to $d\bar{d}$-pairs would be suppressed by Pauli principle because of the presence of two valence u quarks but of only one valence d quark. Assuming this point of view, the experimental result (2) naturally leads to the conclusion that quantum statistical effects play a sensible role in parton dynamics and that, in particular, parton distribution functions are affected by them. In this picture one may also easily account for the known dominance at high x of u-quarks over d-quarks, whose characteristic signature is the fast decreasing of the ratio $F_2^n(x)/F_2^p(x)$ in this regime.

In recent papers [5] this idea has been developed, succeeding in making reasonable assumptions for various polarized parton distributions in terms of unpolarized ones, explaining the observed violation of Ellis-Jaffe sum rule [6], and giving a possible solution to the spin crisis problem [7]. Finally, using Fermi-Dirac or Bose-Einstein inspired form for parton distribution functions, a rather good agreement has been obtained with the experimental data on structure functions [8].

The aim of this letter is to study the role of quantum statistical effects, namely Pauli exclusion principle and induced gluon emission, in the Q^2-evolution of parton distributions. We will show that there is a quite close analogy between the well-known

Altarelli-Parisi (A-P) evolution equations [9], in leading-log approximation, and a set of Boltzmann equations written for a dilute system of particles. This analogy will guide us in finding the generalized scaling when also quantum statistics have been taken into account.

As well-known, the logarithmic dependence on Q^2 of the parton distribution momenta, predicted in the framework of perturbative QCD, has a simple and beautiful interpretation in terms of evolution equations for parton distribution functions. At leading-log level, the A-P equations can be written in the following way

$$\frac{d}{dt}p_A(x,t) = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\sum_B p_B(y,t)P_{AB}\left(\frac{x}{y}\right) \quad , \tag{4}$$

where $t = \ln(Q^2/\mu^2)$, μ is some renormalization scale and $p_A(x,t)$ denote the parton distribution functions ($A, B =$quarks, antiquarks and gluons). By defining

$$\tau \equiv \frac{1}{2\pi b}\ln\left[\frac{\alpha(\mu^2)}{\alpha(t)}\right] \quad , \tag{5}$$

with $b \equiv (33-2n_f)/(12\pi)$ (n_f is the number of flavours), Eq. (4) becomes

$$\frac{d}{d\tau}p_A(x,\tau) = \int_x^1 \frac{dy}{y}\sum_B p_B(y,\tau)P_{AB}\left(\frac{x}{y}\right) \quad . \tag{6}$$

Note that the dependence on τ of r.h.s. of (6) comes only through $p_B(y,\tau)$.
In Eqs. (4) and (6), $P_{AB}(x/y)$ stand for the splitting functions evaluated by using standard *equivalent parton method*. They correspond to the probability for the elementary three-body processes to occur in which a parton with momentum fraction x is produced by a parton with higher fraction $y = x/z$.

The simple microscopical interpretation of Eq. (6) is, as well-known, that the τ dependence of p_A distributions is induced by these processes considering them as occurring in the vacuum.
However, it is physically reasonable to imagine that this picture has to be modified for sufficiently low x; in this regime the nucleons are filled with a large number of quark-antiquark pairs and gluons (the sea) and thus, to take into account in the correct way the presence of this large number of partons, the decays $A \to B + C$ should be considered in presence of a surrounding plasma of both Fermi and Bose particles. Corrections induced by quantum statistical effects to the scaling behaviour dictated by (6) are therefore generally present, and in particular we expect that:

a) Pauli blocking will suppress the production of quarks and antiquarks with fraction x corresponding to $filled$ levels;

b) the gluon emission probability through bremsstrählung processes, considered in the standard picture leading to A-P equations, will be enhanced by the contribution of induced-emission in presence of a rather relevant number of gluons in the sea.

These effects would favour the production of gluon-quark pairs with larger values of x for the quarks and a smaller one for the gluon. Moreover the gluon conversion processes in $q-\bar{q}$ pairs are expected to be reduced.

In statistical mechanics all these effects are simply included by multiplying the amplitudes modulus squared of the relevant processes by the factors $1-f$ or $1+f$ for each Fermi or Bose particle in the final state, with f denoting the particle distribution functions without any level-density factor. In equilibrium conditions these f reach the standard stationary Fermi-Dirac or Bose-Einstein form, while in general they depend on time. Thus, it is reasonable to expect that similar factors should be introduced in the A-P equations.

To go further on this point we will show, as already mentioned, that A-P evolution equations can be formally viewed as Boltzmann transport equations for parton distributions when they satisfy classical statistics (dilute system).

As well-known, the Boltzmann set of equations describes the evolution to equilibrium states of systems composed by many particles of several species (i specie-index $i=1,..,n$) mutually interacting [10]. Assuming, for the following applications, monodimensional dynamics for particles we can define the numerical distribution functions as

$$n_i(\epsilon,t) \equiv g_i(\epsilon) f_i(\epsilon,t) \quad , \tag{7}$$

with ϵ denoting the energy, $f_i(\epsilon,t)$ the statistical functions (they recover the usual Bose/Einstein or Fermi/Dirac at the thermal equilibrium), and $g_i(\epsilon)$ the level-densities (weights) corresponding to ϵ. These last quantities should be fixed from the beginning, by studying the hamiltonian of the total system. From (7) follows the expression for the total number of i-particles

$$N_i(t) = \int d\epsilon \; g_i(\epsilon) f_i(\epsilon,t) \quad . \tag{8}$$

By using Eq. (7), the Boltzmann equations can be cast in the following form

$$\mathcal{L}\; n_i = C_i[\mathbf{f},\mathbf{g}] = C_i^+[\mathbf{f},\mathbf{g}] - C_i^-[\mathbf{f},\mathbf{g}] \qquad\qquad i=1,....,n \quad , \tag{9}$$

where $\mathbf{f} \equiv (f_1, ..., f_n)$, $\mathbf{g} \equiv (g_1, ..., g_n)$, $\mathcal{L}$ is the Liouville operator, and $C_i[\mathbf{f}, \mathbf{g}]$ is the so called collisional integral for the i-th particle specie. The latter is given by a thermal average of all possible processes which change the density of the i-th specie. Notice that in Eq. (9) we have defined $C_i^+[\mathbf{f}, \mathbf{g}]$ and $C_i^-[\mathbf{f}, \mathbf{g}]$ as the contributions corresponding to the interaction processes which create or destroy the i-th particle specie respectively. For simple three body processes $A \to B + C$, $B \to A + C$, if we are interested in describing the modification of B population, the corresponding terms in $C_B[\mathbf{f}, \mathbf{g}]$ are the following

$$
\begin{aligned}
C_B^+[\mathbf{f}, \mathbf{g}] - C_B^-[\mathbf{f}, \mathbf{g}] = 2\pi \int\!\!\int d\epsilon_A \; d\epsilon_C \; \Big\{ & |\mathcal{M}(A \to B + C)|^2 \; \delta(\epsilon_A - \epsilon_B - \epsilon_C) \\
& \times \, n_A(\epsilon_A, t) \; g_B(\epsilon_B) \; [1 \pm f_B(\epsilon_B, t)] \; g_C(\epsilon_C) \; [1 \pm f_C(\epsilon_C, t)] \Big\} \\
-2\pi \int\!\!\int d\epsilon_A \; d\epsilon_C \; \Big\{ & |\mathcal{M}(B \to A + C)|^2 \; \delta(\epsilon_B - \epsilon_A - \epsilon_C) \\
& \times \, n_B(\epsilon_B, t) \; g_A(\epsilon_A) \; [1 \pm f_A(\epsilon_A, t)] \; g_C(\epsilon_C) \; [1 \pm f_C(\epsilon_C, t)] \Big\}
\end{aligned}
\tag{10}
$$

where $\mathbf{f} \equiv (f_A(\epsilon_A, t), f_B(\epsilon_B, t), f_C(\epsilon_C, t))$, $\mathbf{g} \equiv (g_A(\epsilon_A), g_B(\epsilon_B), g_C(\epsilon_C))$, $|\mathcal{M}|^2$ are the squared moduli of transition amplitudes and the sign in the final state factors is positive/negative depending on the bosonic/fermionic nature of particles. In the limit of very small f_i one has $(1 \pm f_i) \sim 1$, and assuming free particle states ($g_i = const$) the collisional term for very dilute systems is recovered.

Assuming n_f different flavours for quarks ($j = 1, ..., n_f$) with elicity states ($\lambda = +, -$), we can rewrite the set of equations (4) for polarized quarks ($q_{j\lambda}$), antiquarks ($\bar{q}_{j\lambda}$), and not-polarized gluons (G) distribution functions in terms of two-dimensional integrals

$$
\frac{d}{d\tau} q_{j\lambda}(x, \tau) = \int_0^1 \int_0^1 dydz \; \delta(x - yz) \; [P_{qq}(z) q_{j\lambda}(y, \tau) + \frac{1}{2} P_{qG}(z) G(y, \tau)] \quad , \tag{11}
$$

$$
\frac{d}{d\tau} \bar{q}_{j\lambda}(x, \tau) = \int_0^1 \int_0^1 dydz \; \delta(x - yz) \; [P_{qq}(z) \bar{q}_{j\lambda}(y, \tau) + \frac{1}{2} P_{qG}(z) G(y, \tau)] \quad , \tag{12}
$$

$$
\begin{aligned}
\frac{d}{d\tau} G(x, \tau) &= \int_0^1 \int_0^1 dydz \; \delta(x - yz) \; \Big\{ P_{GG}(z) G(y, \tau) \\
&+ \sum_{j=1}^{n_f} \sum_{\lambda=+,-} P_{Gq}(z) \Big[q_{j\lambda}(y, \tau) + \bar{q}_{j\lambda}(y, \tau) \Big] \Big\} \quad .
\end{aligned}
\tag{13}
$$

We have assumed for polarized gluon distributions that $G_+(x, \tau) \sim G_-(x, \tau) \sim G(x, \tau)/2$: we will comment on this point later. Note that in the previous expressions the

integrating-variables y and z vary from 0 to 1, i.e. to the maximum available energy properly normalized. Starting from the first of above equations, Eq. (11), we notice that if we formally regard τ as a *time* parameter, r.h.s. of (11) represents the $C^{+}_{j\lambda}$ collisional term of a Boltzmann equation, written for particles obeying to a monodimensional dynamics. This equation is given in terms of numerical distribution functions $q_{j\lambda}(x,\tau)$, $G(x,\tau)$ and of the probabilities for the elementary processes $P_{qq}(z)$ and $P_{qG}(z)$ (the final particles are assumed to be free which means no presence of extra-g terms corresponding to them). It is worth-while pointing out that in the infinite-momentum frame, where the parton picture is well-defined, all transverse dynamics can be safely neglected (it has been already integrated out, indeed) and thus the description is monodimensional. The δ-function in (11) is just the longitudinal momentum conservation in the three body interaction process.
However, to complete the analogy between (11) and the corresponding Boltzmann equation we still have the difficulty that no $C^{-}_{j\lambda}$ terms are included in (11): these correspond to decays of the x-momentum quark in parton pairs and would only depend on $q_{j\lambda}(x,\tau)$ if all statistical factors for final states are neglected. The splitting functions involved in this case are two: $P_{qq}(z)$ and $P_{Gq}(z)$, being respectively, the emission probability of a quark or a gluon with fraction $y = zx$. Notice that in $C^{-}_{j\lambda}$ terms the momentum conservation leads to a definition of z which is simply the inverse of the one in $C^{+}_{j\lambda}$. Thus, the $C^{-}_{j\lambda}$ collisional integral would be

$$C^{-}_{j\lambda}\left[\mathbf{q}(x,\tau),G(x,\tau)\right] = q_{j\lambda}(x,\tau)\int_0^1\int_0^1 dydz\;\delta\left(x-\frac{y}{z}\right)\;\left[P_{qq}(z)+P_{Gq}(z)\right] \quad , \tag{14}$$

with $\mathbf{q} = (q_{1+},..,q_{n_f+},q_{1-},..,q_{n_f-})$, and by integrating over y we finally get

$$C^{-}_{j\lambda}\left[\mathbf{q}(x,\tau),G(x,\tau)\right] = q_{j\lambda}(x,\tau)\int_0^1 dz\;z\;\left[P_{qq}(z)+P_{Gq}(z)\right] = 0 \quad , \tag{15}$$

where in (15) we have used the integral constraint following from momentum conservation in quark splitting. Therefore from (15) since the $C^{-}_{j\lambda}$ contribution vanishes, we deduce that A-P evolution equation for $q_{j\lambda}$ distribution can be consistently viewed as a Boltzmann equation in which all statistical factors corresponding to final particle are neglected (dilute system). Similar considerations can be repeated for the antiquarks evolution equations (12).

A completely analogous result also holds for gluon distribution, whose evolution is dictated by (13). In this case, by writing the corresponding Boltzmann *inspired*

equation, one gets

$$\frac{d}{d\tau}G(x,\tau) = \int_0^1 \int_0^1 dy\ dz\ \delta(x-yz)\Big\{P_{GG}(z)G(y,\tau) + \sum_{j=1}^{n_f}\sum_{\lambda=+,-} P_{Gq}(z)$$

$$\times\Big[q_{j\lambda}(y,\tau)+\bar{q}_{j\lambda}(y,\tau)\Big]\Big\} - G(x,\tau)\int_0^1\int_0^1 dy\ dz\ \delta\left(x-\frac{y}{z}\right)[P_{GG}(z)+2n_f P_{qG}(z)]$$

$$= \int_x^1 \frac{dy}{y}\left\{P_{GG}\left(\frac{x}{y}\right)G(y,\tau) + \sum_{j=1}^{n_f}\sum_{\lambda=+,-} P_{Gq}\left(\frac{x}{y}\right)\Big[q_{j\lambda}(y,\tau)+\bar{q}_{j\lambda}(y,\tau)\Big]\right\}, \quad (16)$$

which is just the A-P equation for gluon distributions. The C_G^- term still vanishes due to the momentum conservation constraint

$$\int_0^1 dz\ z[P_{GG}(z)+2n_f P_{qG}(z)] = 0 \quad . \quad (17)$$

Let us briefly summarize our results till this point: we have shown that A-P evolution equations can be formally regarded as a set of Boltzmann equations for parton distribution functions, in which the Liouville operator takes the simple form of derivative with respect to τ scaling variable. It is worth to notice that the absence of any external force in the regime of high transferred Q^2 is of course compatible with this expression for $\mathcal{L}$. The analogy holds under the hypothesis that quarks and gluons form a very dilute system in the nucleons, so the statistical factors for final particles in the interaction processes can be neglected. Starting from this equivalence and urged from the idea that, instead, quantum statistics would play a role in parton dynamics, it is now easy to generalize A-P equations to a set of generalized scaling equations where Pauli exclusion principle and gluon stimulated emission processes can be taken into account in a simple way.

To this aim, as in equation (10), one should introduce in the collisional integrals the $(1 \pm f_i)$ factors, and thus, as long as the statistical effects are taken into account, the factorization of $q_{j\lambda}$, $\bar{q}_{j\lambda}$ and G as reported in Eq. (7) becomes necessary.
In the same spirit of (7), we will write the quark, antiquark and gluon distributions as

$$\begin{aligned} q_{j\lambda}(x,\tau) &= g_{j\lambda}(x)\ f_j^\lambda(x,\tau) \quad , & (18)\\ \bar{q}_{j\lambda}(x,\tau) &= \bar{g}_{j\lambda}(x)\ \bar{f}_j^\lambda(x,\tau) \quad , & (19)\\ G(x,\tau) &= g_G(x)\ f_G(x,\tau) \quad , & (20) \end{aligned}$$

where $g_{j\lambda}(x)$, $\bar{g}_{j\lambda}(x)$ and $g_G(x)$ are weight functions, whereas $f_j^\lambda(x,\tau)$, $\bar{f}_j^\lambda(x,\tau)$ and $f_G(x,\tau)$ are purely statistical distributions. The explicit form for g-functions, which

contains the infrared divergency at $x = 0$, should be fitted from experimental data, as in [8], or deduced from theoretical expected behaviour, like, for example, Regge theory. We stress that the factorized form (18)-(20), in particular the hypothesis that the singular functions $g_{j\lambda}$, $\bar{g}_{j\lambda}$ and g_G do not depend on τ is compatible with predictions of both Regge theory and QCD for the behaviour of parton distributions at the end-point $x = 0$; as well-known in this regime one has

$$p_A(x, Q^2) \sim \xi_A(Q^2) x^{-\alpha_A} \quad , \tag{21}$$

with α_A which does not depend on Q^2, at least for large Q^2 [11].
Remarkably, a parameterization similar to (18)-(20) has been already successfully proposed on phenomenological basis in [8], to fit all the available measurements on parton distributions at fixed Q^2, assuming for them a thermal-equilibrium form

$$q_{j\lambda}(x) = A\, x^{-\alpha} \left[\exp\left(\frac{x - x_{j\lambda}}{\tilde{x}}\right) + 1\right]^{-1} \quad , \tag{22}$$

$$\bar{q}_{j\lambda}(x) = A\, x^{-\alpha} \left[\exp\left(\frac{x - \bar{x}_{j\lambda}}{\tilde{x}}\right) + 1\right]^{-1} \quad , \tag{23}$$

$$G(x) = \frac{16}{3}\, A\, x^{-\alpha} \left[\exp\left(\frac{x - x_G}{\tilde{x}}\right) - 1\right]^{-1} \quad , \tag{24}$$

where $x_{j\lambda}$, $\bar{x}_{j\lambda}$, and x_G represent the *thermodynamical potentials*, and $\tilde{x}$ plays the role of the *temperature*. It is worth-while to point out that, in the framework of a formal connection between the A-P evolution equations and Boltzmann equations, these results [8] are a straight consequence of the above analogy, which predicts thermal-equilibrium-like solutions for parton distribution functions for sufficiently high values of Q^2.

Within the factorized expression (18)-(20) the final state factors are written in the form $1 - f_j^\lambda$, $1 - \bar{f}_j^\lambda$ and $1 + f_G$ for quarks, antiquarks and gluons respectively.

We are now able to introduce a set of generalized scaling equations for quarks and gluons. Here we will consider for simplicity the case in which the gluons are supposed to not have a significant net polarization in the nucleons with respect to the one carried by quarks. We will assume, therefore $G_+(x, \tau) \sim G_-(x, \tau) \sim G(x, \tau)/2$. It should be pointed out that this approximation is consistent with the results obtained in [5] and [8], where it is argued that still Pauli principle plays an essential role to generate the polarization of the quark sea, in the same approach therefore adopted here.
It is, instead, less satisfactory in the framework of the different interpretation of the violation of Ellis-Jaffe sum rule based on the axial-vector current anomaly [12]. This

latter case, in fact, would require a very large gluon polarization, i.e. $\Delta G = G_+ - G_- \sim 3 \div 4$. Notice however that, as shown in [8], gluons are expected to be more numerous than quarks, due to their Bose nature, so in any case one has $\Delta G/G << \Delta q/q$, which supports our approximation.

By helicity conservation at the quark-gluon vertex, it is easily seen that evolution equations for polarized quark distribution functions get the following form

$$
\begin{aligned}
\frac{d}{d\tau} q_{j\lambda}(x,\tau) &= \int_x^1 \frac{dz}{z} \left\{ P_{qq}(z)\; q_{j\lambda}\left(\frac{x}{z},\tau\right) \left[1 - f_j^\lambda(x,\tau)\right] \left[1 + \frac{1}{2} f_G\left(x\left(\frac{1}{z}-1\right),\tau\right)\right] \right. \\
&+ \left. \frac{1}{2} P_{qG}(z)\; G\left(\frac{x}{z},\tau\right) \left[1 - f_j^\lambda(x,\tau)\right] \left[1 - \bar{f}_j^{-\lambda}\left(x\left(\frac{1}{z}-1\right),\tau\right)\right] \right\} \\
&- q_{j\lambda}(x,\tau) \int_0^1 z\; dz \left\{ P_{qq}(z)\; \left[1 - f_j^\lambda(xz,\tau)\right] \left[1 + \frac{1}{2} f_G\left(x\left(1-z\right),\tau\right)\right] \right. \\
&+ \left. P_{Gq}(z)\; \left[1 + \frac{1}{2} f_G(xz,\tau)\right] \left[1 - f_j^\lambda\left(x\left(1-z\right),\tau\right)\right] \right\} \quad . \qquad (25)
\end{aligned}
$$

The equations for antiquarks are easily obtained by the previous one by substituting $q_{j\lambda} \leftrightarrow \bar{q}_{j\lambda}$ and $f_j^\lambda \leftrightarrow \bar{f}_j^\lambda$. Similarly for the gluon distribution $G(x,\tau)$ one has

$$
\begin{aligned}
\frac{d}{d\tau} G(x,\tau) &= \int_x^1 \frac{dz}{z} \left\{ P_{GG}(z)\; G\left(\frac{x}{z},\tau\right) \left[1 + \frac{1}{2} f_G(x,\tau)\right] \left[1 + \frac{1}{2} f_G\left(x\left(\frac{1}{z}-1\right),\tau\right)\right] \right. \\
&+ \sum_{j=1}^{n_f} \sum_{\lambda=+,-} P_{Gq}(z)\; \left[1 + \frac{1}{2} f_G(x,\tau)\right] \left\{ q_{j\lambda}\left(\frac{x}{z},\tau\right) \left[1 - f_j^\lambda\left(x\left(\frac{1}{z}-1\right),\tau\right)\right] \right. \\
&+ \left. \left. \bar{q}_{j\lambda}\left(\frac{x}{z},\tau\right) \left[1 - \bar{f}_j^\lambda\left(x\left(\frac{1}{z}-1\right),\tau\right)\right] \right\} \right\} \\
&- G(x,\tau) \int_0^1 z\; dz \left\{ P_{GG}(z)\; \left[1 + \frac{1}{2} f_G(xz,\tau)\right] \left[1 + \frac{1}{2} f_G\left(x\left(1-z\right),\tau\right)\right] \right. \\
&+ \frac{1}{2} \sum_{j=1}^{n_f} \sum_{\lambda=+,-} P_{qG}(z)\; \left\{ \left[1 - f_j^\lambda(xz,\tau)\right] \left[1 - \bar{f}_j^{-\lambda}\left(x\left(1-z\right),\tau\right)\right] \right. \\
&+ \left. \left. \left[1 - \bar{f}_j^\lambda(xz,\tau)\right] \left[1 - f_j^{-\lambda}\left(x\left(1-z\right),\tau\right)\right] \right\} \right\} \quad . \qquad (26)
\end{aligned}
$$

Several comments can be made on the above expressions. First of all we notice that the inverse decay processes, the ones contained in the C_i^- collisional integral, contribute to the scaling behaviour of parton distribution functions with terms quadratic in the $p_A(x,\tau)$ at least. This is a consequence of our interpretation of A-P equations as *transport equations*. As already shown these terms vanish in the limit of a very dilute system.

The generalized equations predict also a different, more complicated, evolution for momenta. By taking Mellin transform of both sides of (25) and (26), in fact, one sees that the standard scaling behaviour should be corrected by terms quadratic and cubic in distribution functions, which are not simply products of momenta of quarks and gluon densities.

Finally, as for the standard A-P equations, the scaling behaviour for unpolarized quark distributions can be obtained by simply considering the sum $q_j(x,\tau) = q_{j+}(x,\tau) + q_{j-}(x,\tau)$ (the same holds for antiquarks). Notice, however, that since the introduction of final state statistical factors spoils the linearity of the equations, the evolution of $q_j(x,\tau)$ will depend on both the polarized distribution functions and not simply on their sum.

We are now at some concluding remarks. We have stressed the point that, as some experimental results suggest, the Fermi or Bose nature of partons could sensibly manifest itself in observable quantities in deep inelastic scattering on nucleons. This idea already successfully applied in [5] and [8] mostly motivates our paper. In particular it seems to us quite natural that quantum statistics may modify the scaling behaviour of parton distribution functions for rather small x and high Q^2; in this region the sea becomes dominant and thus bremsstrählung processes, responsible at leading-log level for scaling breaking, are likely supposed to occur in presence of a *gas* of partons. In this case Pauli blocking and gluon stimulated emission play, in general, a sensible role in parton distribution evolution.

We have introduced both this statistical effects obtaining the generalized scaling, starting from the observation that a quite close and intriguing analogy seems to hold between A-P equations and a set of Boltzmann transport equations for partons. In our approach to nucleons as *statistical* systems, this fact is of course welcome and expected. Pursuing this formal analogy it seems to us very fascinating the fact that the scale variable Q^2 can be interpreted in some sense as a *time* parameter. The physical significance of this point, if any, should be deeper understood. It is also rather interesting to stress that, from this point of view, one would naturally expect, in the spirit of Boltzmann H theorem, that the normalized parton distributions $f_{j\lambda}(x,\tau)$, $\bar{f}_{j\lambda}(x,\tau)$ and $f_G(x,\tau)$ should approach stationary Fermi and Bose expressions as Q^2 increases. Remarkably, these conclusions seem to agree with the phenomenological results obtained in [8].

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